Chapter 6 – Additional Topics in Trig

TOPIC 1: LAW OF SINES – P. 712

- HOMEWORK: P. 721 #1-16 EVEN, P. 721 #47-51

TOPIC 2: AMBIGUOUS CASE/AREA OF A TRIANGLE – P. 718

- HOMEWORK: P. 721 #17-38 EVEN

TOPIC 3: LAW OF COSINES – P. 724

- HOMEWORK: P. 730 #9-30 EVEN, P. 731 # 42-45

TOPIC 4: POLAR COORDINATES/CONVERSIONS (2 LESSONS) – P. 733

- HOMEWORK: WORKSHEET & P. 742 & 743 #33-74 EVEN

TOPIC 5: GRAPHS OF POLAR EQUATIONS (2 LESSONS) – P. 745

- HOMEWORK: WORKSHEET & P. 754 #13-49 EVEN

Name: ____ Date: __________________ Period: _______

Chapter 6: Additional Topics in Trig
Topic 1: Law of Sines

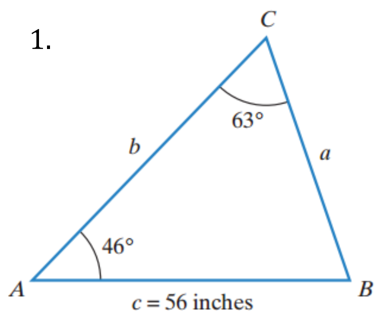
Recall the law of sines from prior courses: The ratio of the length of the side of any triangle to the sine of the angle opposite that side is the same for all three sides of the triangle. We use the following formula:

Be sure to check the pairs you're using!

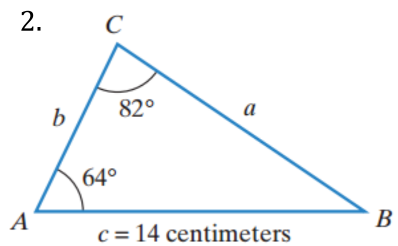
Angle-Side-Angle: When the given information is A-S-A, we can easily find the missing angle, then use the law of sines to solve for all missing sides.

Practice: Solve the triangles below ("Solve the triangle" means to find the measure of all sides and all angles; nearest tenth and nearest degree, respectively.)

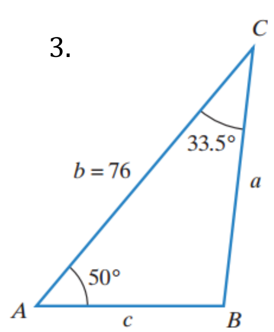
1.



2.



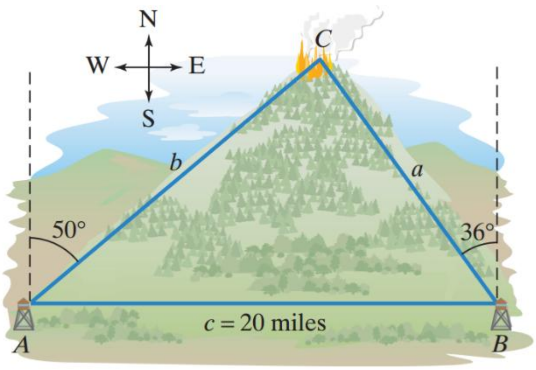
3.



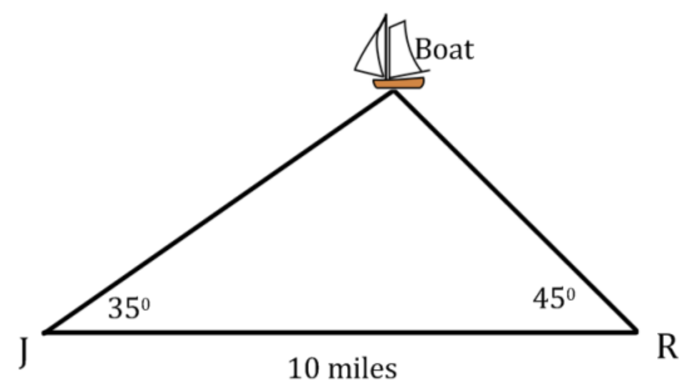
4. Solve triangle ABC if $A = 40°$, $C = 22.5°$, and $b = 12$

Examples applying the law of sines:

5. Two fire-lookout stations are 20 miles apart, with station B directly east of station A. Both stations spot a fire on a mountain to the north. The bearing from station A to the fire is50° east of due-north, and 36° west of due-north from station B, as shown in the diagram. How far is the fire from each station, to the nearest mile?



6. Juan and Romella are standing at the seashore 10 miles apart. The coastline is a straight line between them. Both can see the same ship in the water. The angle between the coastline and the line between the ship and Juan is 35 degrees. The angle between the coastline and the line between the ship and Romella is 45 degrees. To the nearest tenth of a mile, how far is the ship from Juan?



Name: ____ Date: __________________ Period: _______

Chapter 6: Additional Topics in Trig
Topic 1: Homework

In Exercises 9–16, solve each triangle. Round lengths to the nearest tenth and angle measures to the nearest degree.

9. $A = 44°, B = 25°, a = 12$
10. $A = 56°, C = 24°, a = 22$
11. $B = 85°, C = 15°, b = 40$
12. $A = 85°, B = 35°, c = 30$

13. $A = 115°, C = 35°, c = 200$
14. $B = 5°, C = 125°, b = 200$
15. $A = 65°, B = 65°, c = 6$
16. $B = 80°, C = 10°, a = 8$

47. Two fire-lookout stations are 10 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N25°E and the bearing of the fire from station B is N56°W. How far, to the nearest tenth of a mile, is the fire from each lookout station?

48. The Federal Communications Commission is attempting to locate an illegal radio station. It sets up two monitoring stations, A and B, with station B 40 miles east of station A. Station A measures the illegal signal from the radio station as coming from a direction of 48° east of north. Station B measures the signal as coming from a point 34° west of north. How far is the illegal radio station from monitoring stations A and B? Round to the nearest tenth of a mile.

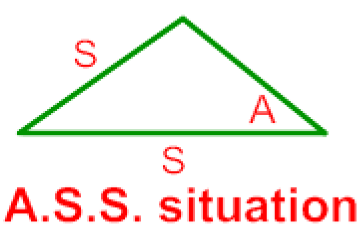
Name: ____ Date: __________________ Period: _______

Chapter 6: Additional Topics in Trig
Topic 2: Ambiguous Case

Angle-Side-Side: When the given information is in a different order, we have to use the law of sines to find all missing pieces of the triangle. Since we will be solving for an angle using the sine function, we must consider that the angle could be obtuse as well. This is called the **Ambiguous Case**.

By definition, the word ambiguous means open to two or more interpretations. Such is the case for certain solutions when working with the Law of Sines.

- **If you are given two angles and one side (ASA or AAS), the Law of Sines will nicely provide you with ONE solution for a missing side.**
- **Unfortunately, the Law of Sines has a problem dealing with SSA. If you are given two sides and one angle (where you must find an angle), the Law of Sines could possibly provide you with one or more solutions, or even no solution.**



Ambiguous Case: when measures of 2 sides and the angle not included between them are given, there may be one Δ, two Δ's, or no Δ possible.

Steps:

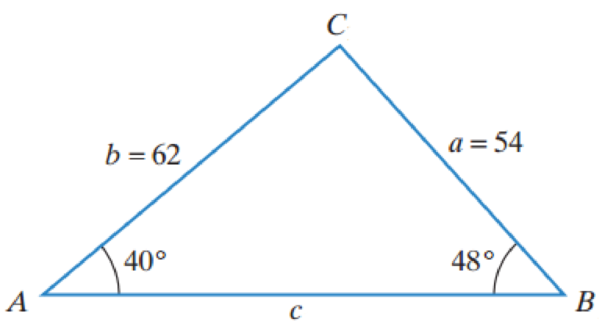
1. Use the law of sines to find the missing angles in Quadrant I and Quadrant II.

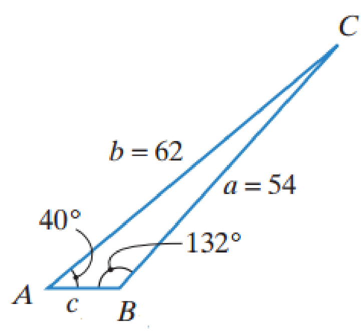
2. Add the given < to both the Quadrant I and Quadrant II angles. (separately)

3. If the sum is less than 180^{o} then a Δ can be formed.

Example: Given triangle ABC with $A = 40°$, $a = 54$ and $b = 62$, solve all possible triangles.

<u>Start by finding all possible values for angle B using the law of sines:</u>





<u>Finish each possible triangle for the remaining side and angle:</u>

Be thoughtful! Sometimes we will learn that one or both triangles aren't possible based on the angle measurements. Remember that all three angles must total exactly 180°, and not more.

Practice: Solve each possible triangle below.

1. Solve triangle ABC if $A = 123°, a = 47$ and $c = 23$

2. Solve triangle ABC if $A = 50°, a = 10$ and $b = 20$

3. Solve triangle ABC if $A = 61°, a = 55$ and $c = 35$

4. Solve triangle ABC if $A = 35°, a = 12$ and $b = 16$

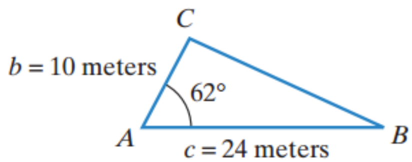
5. Solve triangle ABC if $A = 75°, a = 51$ and $b = 71$

Area of a Triangle: Recall that the area of a triangle is equal to one-half times the lengths of two sides multiplied by the sine of their included angle. We use the following formula:

Letters will change…Be sure you're always using the INCLUDED angle!

Practice: Find the area of the triangles below. Use nearest tenth or nearest degree when necessary

6.



7. Find the EXACT area of a triangle having two sides of lengths 8 yards and 12 yards and an included angle of 135°.

8. Find the area of triangle ABC, to the nearest tenth if $A = 49°, a = 7$ and $B = 112°$

Name: ____ Date: __________________ Period: _______

Chapter 6: Additional Topics in Trig
Topic 2: Homework

In Exercises 17–32, two sides and an angle (SSA) of a triangle are given. Determine whether the given measurements produce one triangle, two triangles, or no triangle at all. Solve each triangle that results. Round to the nearest tenth and the nearest degree for sides and angles, respectively.

17. $a = 20, b = 15, A = 40°$

18. $a = 30, b = 20, A = 50°$

19. $a = 10, c = 8.9, A = 63°$

20. $a = 57.5, c = 49.8, A = 136°$

21. $a = 42.1, c = 37, A = 112°$

22. $a = 6.1, b = 4, A = 162°$

23. $a = 10, b = 40, A = 30°$

24. $a = 10, b = 30, A = 150°$

Name: ______________________________ Date: ________________ Period: ______

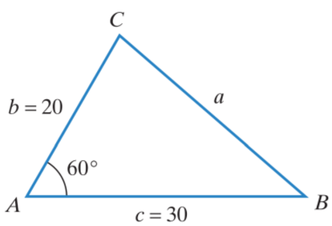
Chapter 6: Additional Topics in Trig
Topic 3: Law of Cosines

Recall the law of cosines from prior courses: The square of a side of a triangle equals the sum of the squares of the other two sides (similar to Pythagorean theorem) minus twice their product times the cosine of their included angle.

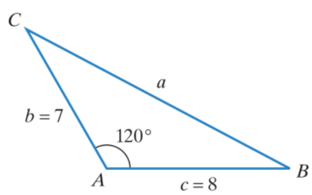
Letters will change…Be sure you're always using the INCLUDED angle!

Practice: Solve the triangles below. Round sides to the nearest tenth and angles to the nearest degree.

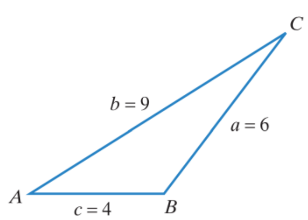
1.



2.



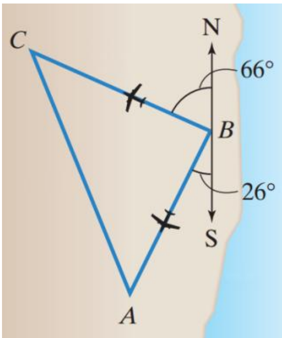
3.



4. Solve triangle ABC if $a = 8$, $b = 10$, and $c = 5$

Examples applying the law of cosines:

5. Two airplanes leave an airport at the same time on different runways. One flies 66° west of north at 325 miles per hour. The other airplane flies 26° west of south at 300 miles per hour. How far apart will the airplanes be after two hours, to the nearest mile?



6. Two airplanes leave an airport at the same time on different runways. One flies directly north at 400 miles per hour. The other airplane flies 75° east of north at 350 miles per hour. How far apart will the airplanes be after two hours, to the nearest mile?

Heron's Formula for Area of a Triangle: When it is the sides of a triangle that is known, the previous formula for area will not be helpful. In this case, we can use Heron's Formula:

Where s is ONE HALF the perimeter of the triangle

Practice: Find the areas below. Round to the nearest tenth when necessary.

7. Find the area of a triangle with side lengths of 12 yards, 16 yards, and 24 yards.

8. Find the area of a triangle with side lengths of 6 feet, 16 feet, and 18 feet.

Name: ____ Date: __________________ Period: ______

Chapter 6: Additional Topics in Trig
Topic 3: Homework

In Exercises 9–24, solve each triangle. Round lengths to the nearest tenth and angle measures to the nearest degree.

9. $a = 5, b = 7, C = 42°$
10. $a = 10, b = 3, C = 15°$
11. $b = 5, c = 3, A = 102°$
12. $b = 4, c = 1, A = 100°$
13. $a = 6, c = 5, B = 50°$
14. $a = 4, c = 7, B = 55°$

In Exercises 25–30, use Heron's formula to find the area of each triangle. Round to the nearest square unit.

25. $a = 4$ feet, $b = 4$ feet, $c = 2$ feet
26. $a = 5$ feet, $b = 5$ feet, $c = 4$ feet
27. $a = 14$ meters, $b = 12$ meters, $c = 4$ meters

Name: ____ Date: ____________________ Period: _______

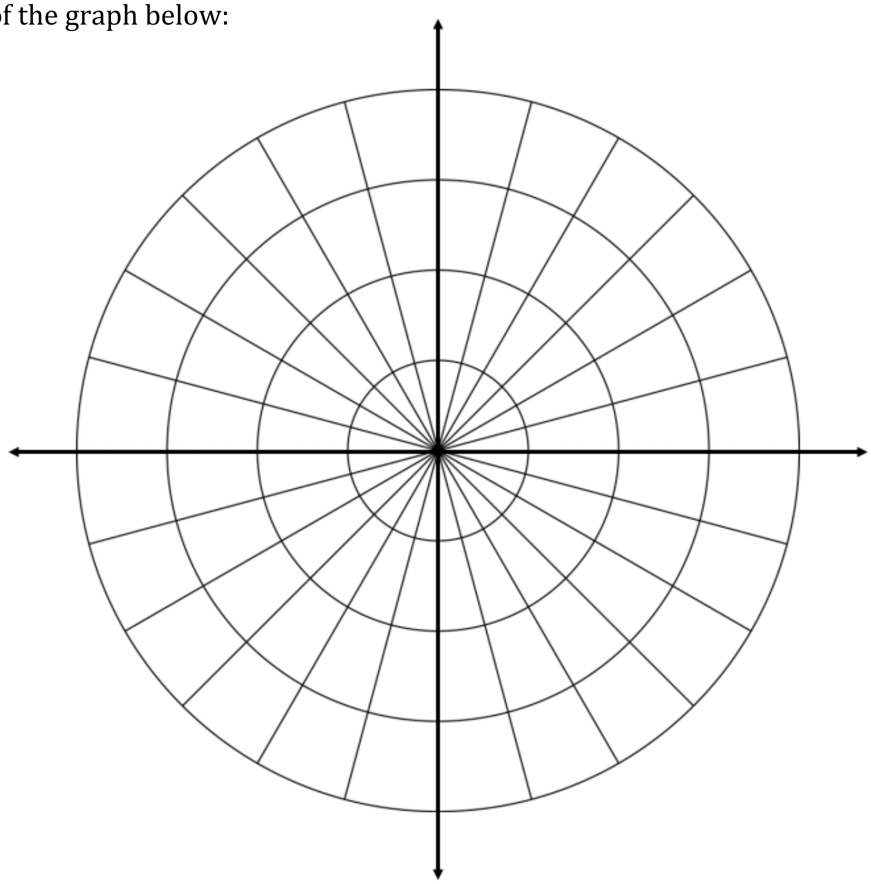
Chapter 6: Additional Topics in Trig
Topic 4: Polar Coordinates/Conversions

Polar coordinates are given as a point $(\boldsymbol{r}, \boldsymbol{\theta})$ where:

r is the directed distance from the pole — *Pole: Think 'origin'*

θ is an angle opening from the polar axis — *Polar Axis: Think 'Initial Ray' or 'positive x-axis'*

Label all key features of the graph below:

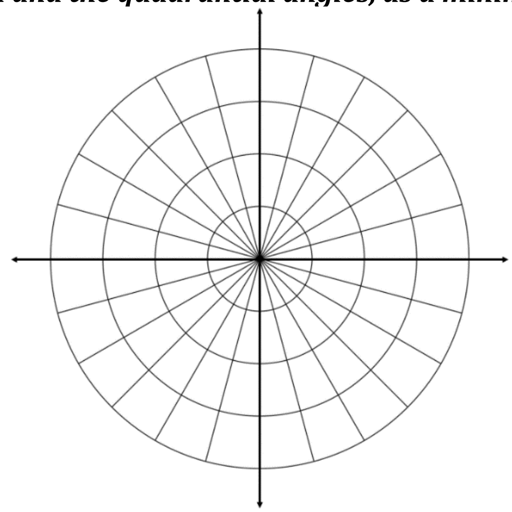


Note: If a question or polar coordinate is presented in radians, then the graph must be labeled in radians

Plotting points

Typically it is easier to open the angle θ first, then count out the ray.

When using a blank graph, label the r-values in one direction and the quadrantial angles, as a minimum.



Positive values:

- θ opens counterclockwise
- r counts out in a positive direction

Plot the following points on the graph

$A\ (3, 150°)$

$B\ (2, 240°)$

$C\ (3, 300°)$

Negative r-values:

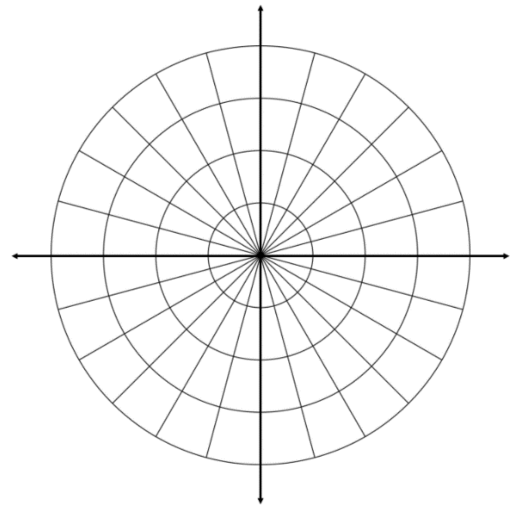
- θ opens counterclockwise
- r counts out in a **negative** direction

Plot the following points on the graph

$A\ (-3, 120°)$

$B\ (-1, 315°)$

$C\ (-4, 210°)$



Negative θ-values:

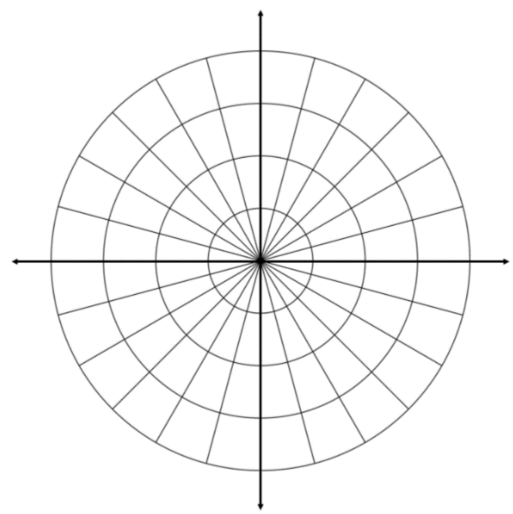
- θ opens **clockwise**
- r counts out in a positive direction

Plot the following points on the graph

$A\ (2, -60°)$

$B\ (4, -135°)$

$C\ (1, -270°)$



Negative r-values and θ-values:

- θ opens **clockwise**
- r counts out in a **negative** direction

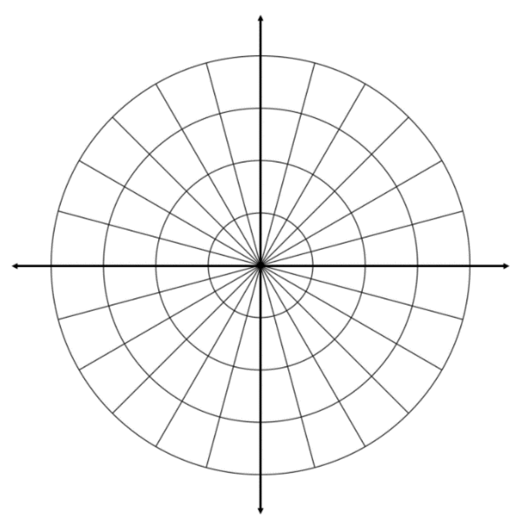
Plot the following points on the graph

$A\ (-2, -45°)$

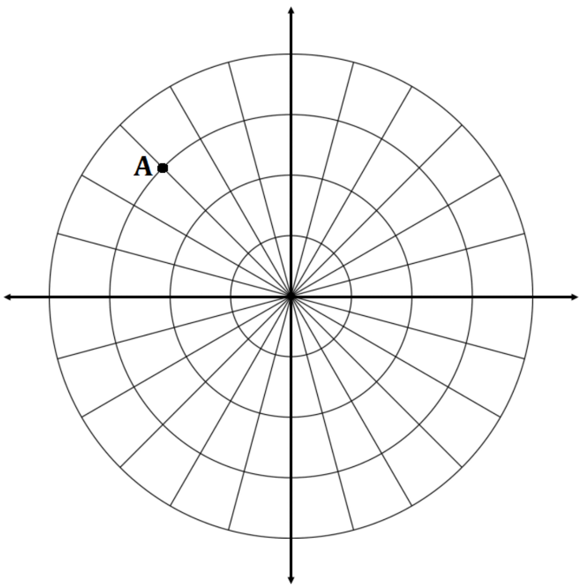
$B\ (-4, -105°)$

$C\ (-3, -270°)$

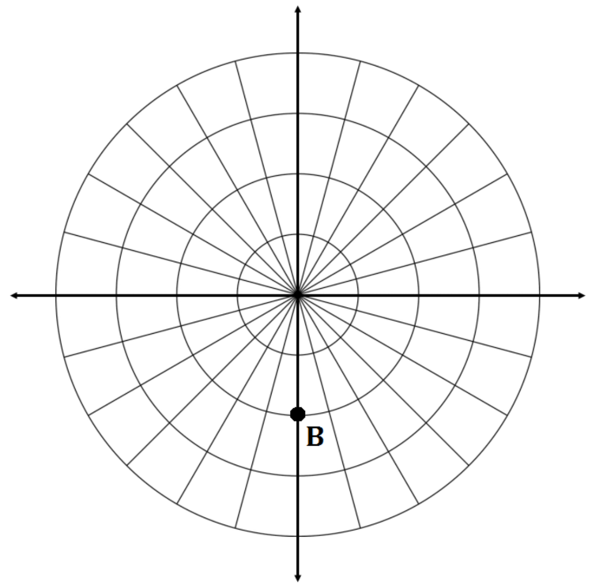
Thinking critically: what's another way you could express each of these points using positive numbers only?



Identify the following points 4 different ways



Given that θ is expressed in degrees



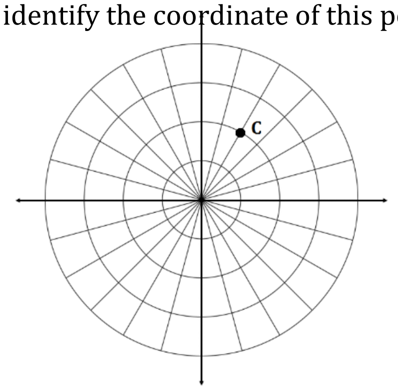
Given that θ is expressed in radians

Degree intervals other than 0 through 360°

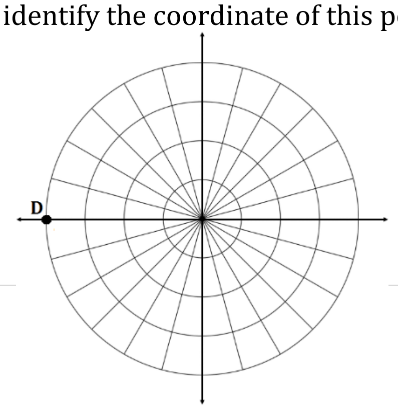
Recall: Coterminal Angles are angles who share the same initial side and terminal sides. Finding coterminal angles is as simple as adding or subtracting 360° or 2π to each angle, depending on whether the given angle is in degrees or radians. The same can be done in the polar coordinate system

<u>**Examples:**</u>

1. A point is plotted on the graph at the right. Given that r is positive, identify the coordinate of this point given each of the following criteria
   - $0 \leq \theta < 360°$
   - $360° \leq \theta < 720°$
   - $-360° \leq \theta < 0$



2. A point is plotted on the graph at the right. Given that r is positive, identify the coordinate of this point given each of the following criteria
   - $0 \leq \theta < 2\pi°$
   - $2\pi \leq \theta < 4\pi$
   - $-2\pi \leq \theta < 0$



Homework:

- Plot and label each of the points below.
- Connect the dots alphabetically.
- Decorate.

D $(1, 15°)$

I $(4, -210°)$

F $(1, 270°)$

J $(-2, 300°)$

C $(4, -330°)$

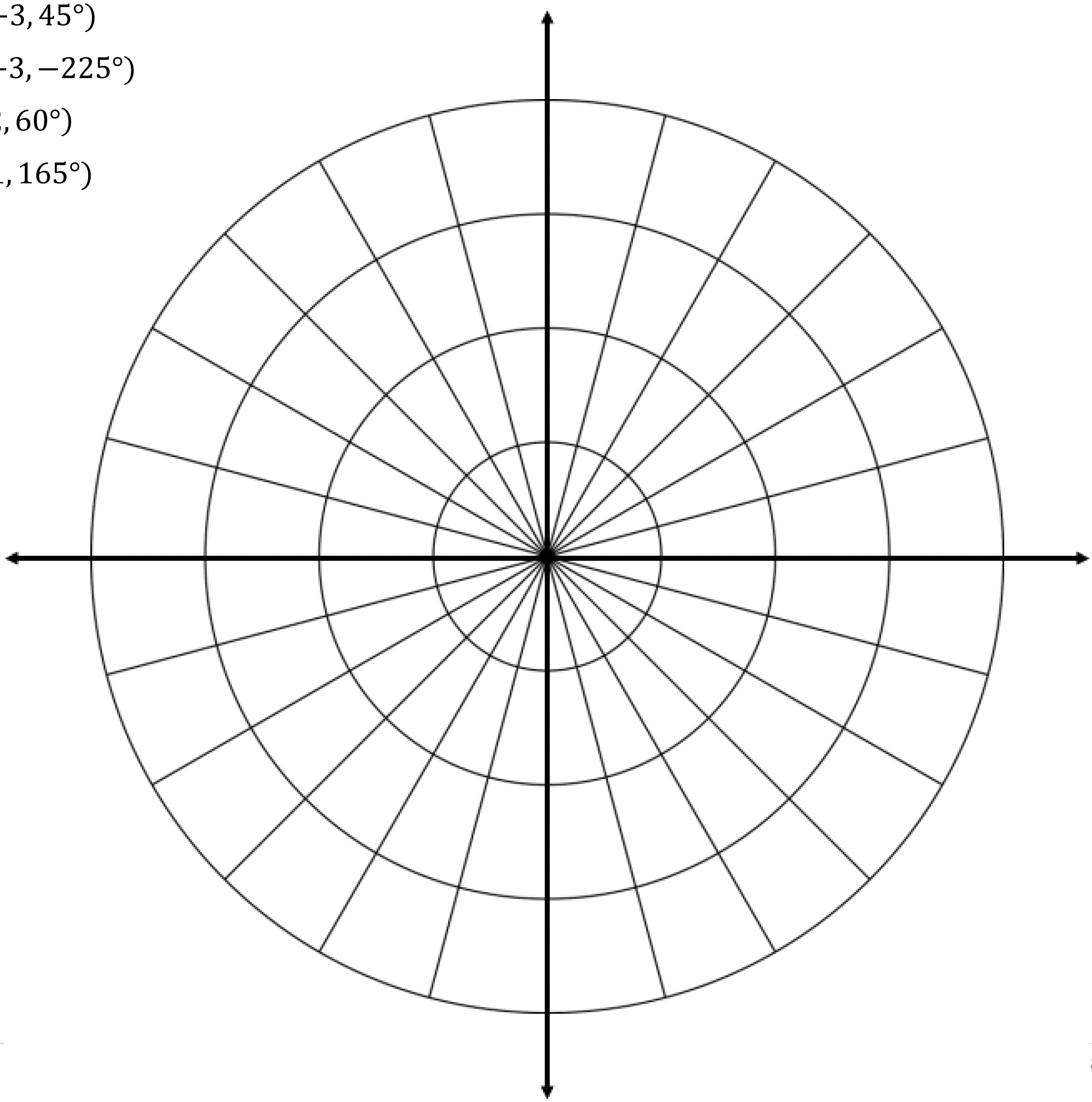
A $(-4, 270°)$

G $(-3, 45°)$

E $(-3, -225°)$

B $(2, 60°)$

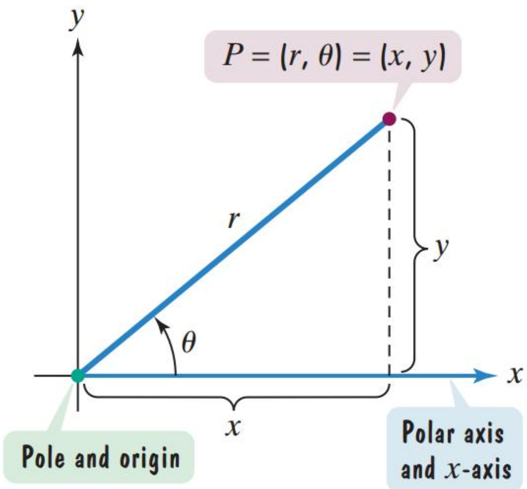
H $(1, 165°)$



Name: ____ Date: __________________ Period: _______

Chapter 6: Additional Topics in Trig
Topic 4: Polar Coordinates/Conversions

CONVERSIONS



Converting from Polar Coordinates (r, θ) to Rectangular Coordinates (x, y)

Using the given values of r and θ, the following relationships are helpful in finding an equivalent (x, y) coordinate:

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$

Gut check! Picture/graph the polar coordinate and the rectangular coordinate. Does it make sense that these are equivalent?

Examples: Find the rectangular coordinates of the points with the following polar coordinates.

1. $\left(2, \frac{3\pi}{2}\right)$

2. $(-8, 60°)$

3. $(-10, 30°)$

4. $(3, \pi)$

Converting from Rectangular Coordinates (x, y) to Polar Coordinates (r, θ)

Using the given values of x and y, the following relationships are helpful in finding an equivalent (r, θ) coordinate:

$$x^2 + y^2 = r^2 \qquad \tan\theta = \frac{y}{x}$$

Examples: Find the polar coordinates of the points with the following rectangular coordinates.

5. $\left(-1, \sqrt{3}\right)$

   Quadrant: *Find r:* *Find θ:* *Answer:*

6. $\left(1, -\sqrt{3}\right)$

7. $(0, -4)$

8. $(-2, 0)$

Converting Equations from Rectangular Form to Polar Form

To convert a rectangular equation in x and y to a polar equation in r and θ:

- Replace x with $r\cos\theta$
- Replace y with $r\sin\theta$.
- Re-solve for r

Examples: Convert each equation to a polar equation.

9. $x + y = 5$

10. $3x - y = 6$

11. $(x-1)^2 + y^2 = 1$

12. $x^2 + (y+1)^2 = 1$

Converting Equations from Polar Form to Rectangular Form

Using the facts from above, we usually need to modify polar equations before we are able to convert them to rectangular equations

Summary of Polar/Rectangular relationships

Examples: Convert each equation to a rectangular equation.

13. $r = 3$

14. $\theta = 45°$

15. $r = \csc\theta$

16. $\theta = \frac{3\pi}{4}$

17. $r = \sec\theta$

18. $r = 4$

19. $r = -6cos$

Name: ____ Date: __________________ Period: _______

Chapter 6: Additional Topics in Trig
Topic 4: Homework

In Exercises 49–58, convert each rectangular equation to a polar equation that expresses r in terms of θ.

49. $3x + y = 7$

50. $x + 5y = 8$

51. $x = 7$

52. $y = 3$

53. $x^2 + y^2 = 9$

54. $x^2 + y^2 = 16$

In Exercises 59–74, convert each polar equation to a rectangular equation. Then use a rectangular coordinate system to graph the rectangular equation.

59. $r = 8$

60. $r = 10$

61. $\theta = \frac{\pi}{2}$

62. $\theta = \frac{\pi}{3}$

63. $r \sin \theta = 3$

64. $r \cos \theta = 7$

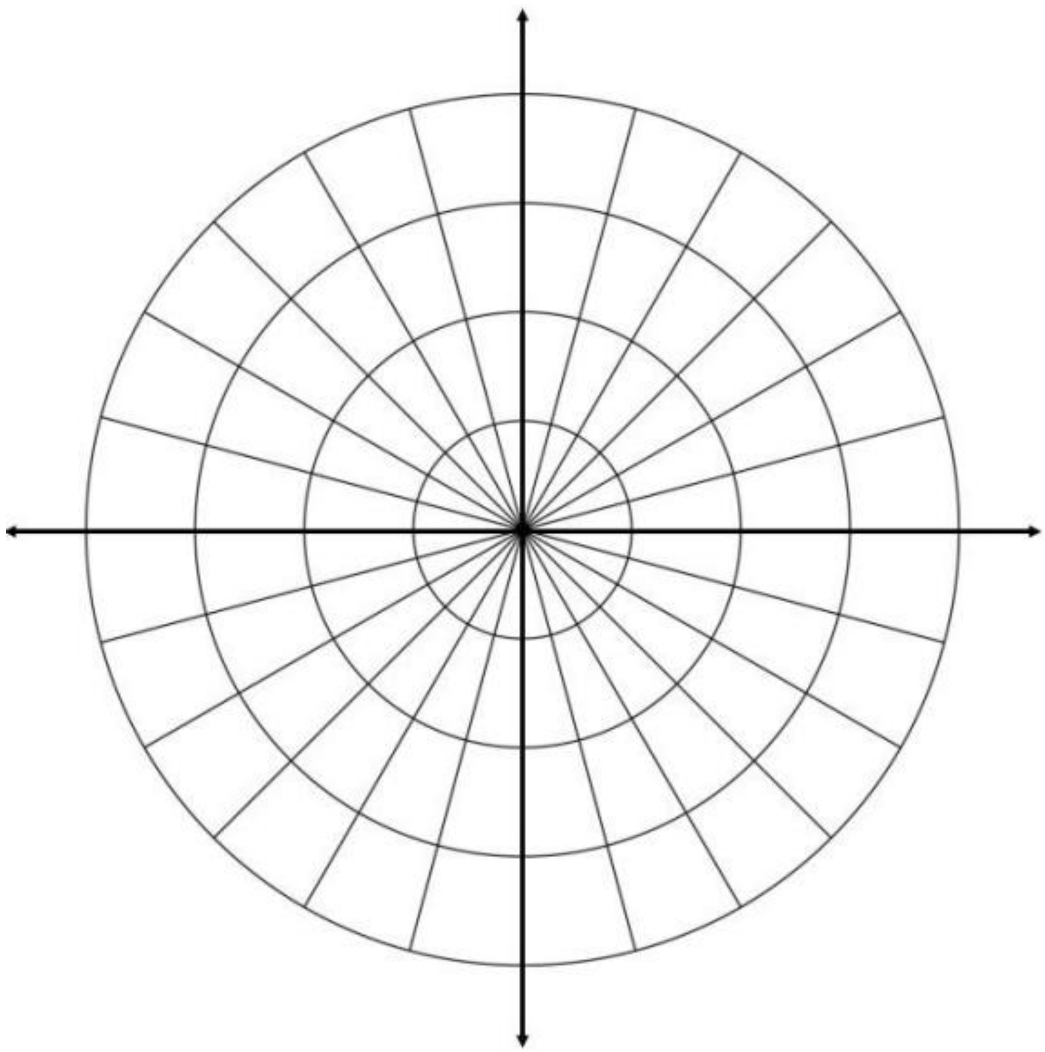
Name: ____ Date: __________________ Period: ______

Chapter 6: Additional Topics in Trig
Topic 5: Graphs of Polar Equations

To graph polar equations, we create a table of values. Similar to trig graphs on rectangular coordinate planes, these equations produce smooth, curved graphs. There should be n breaks or sharp turns. To reduce the chance of an input error, input the equation once into your "Y=" and read values from the table.

Example #1: Create a graph of the equation $r = 4\cos\theta$
Construct a table. Counting by 30's will give us enough points to graph an accurate picture

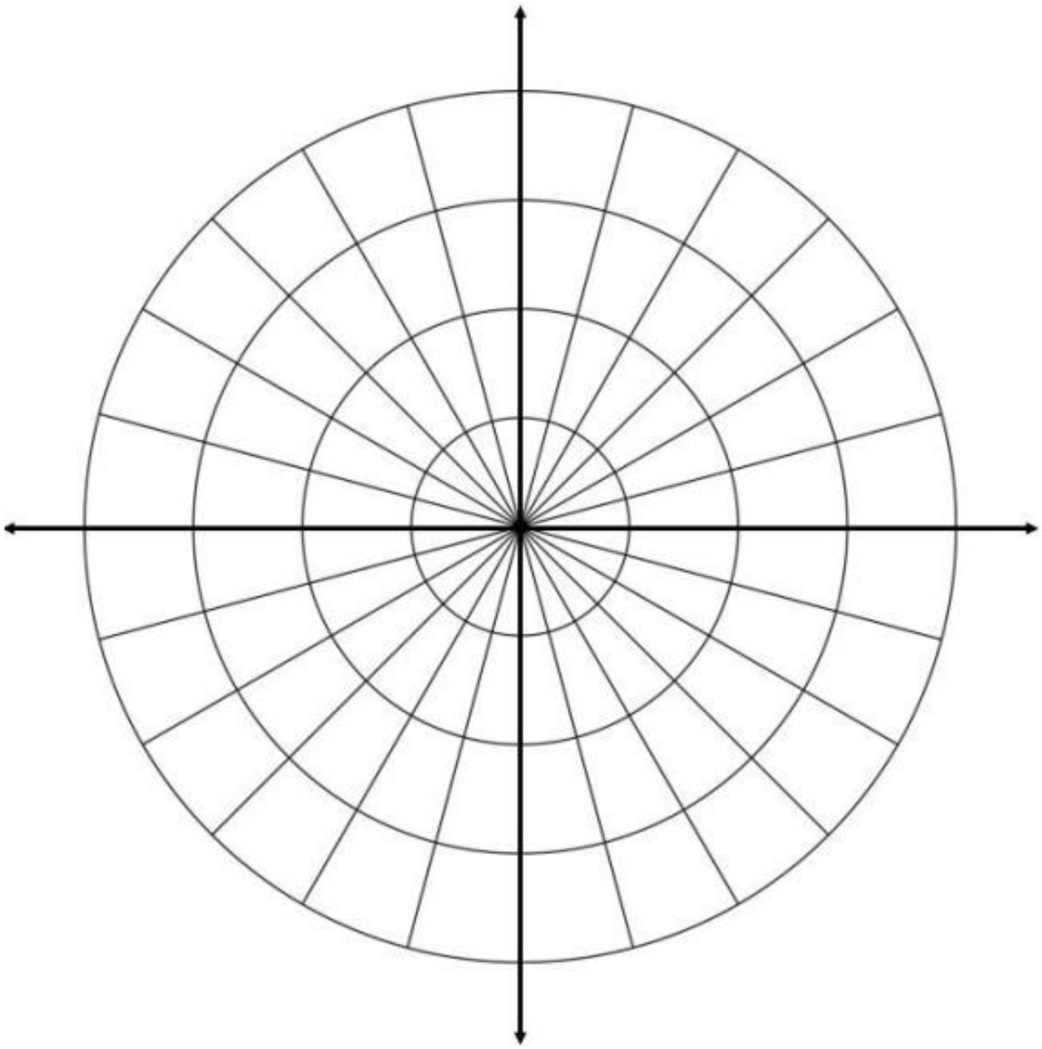
θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$r = 4\cos\theta$													



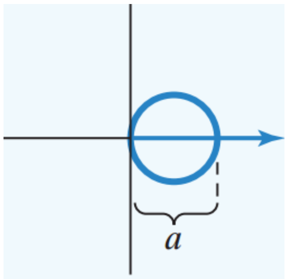
Converting the equation from polar to rectangular will show an equivalent picture:

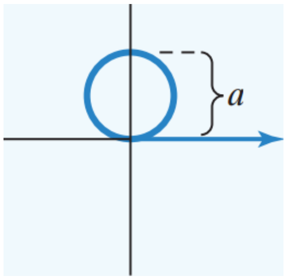
Example #2: Create a graph of the equation $r = 4 \sin \theta$

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$r = 4 \sin \theta$													



Summarize the graphs of the two main trigonometric functions



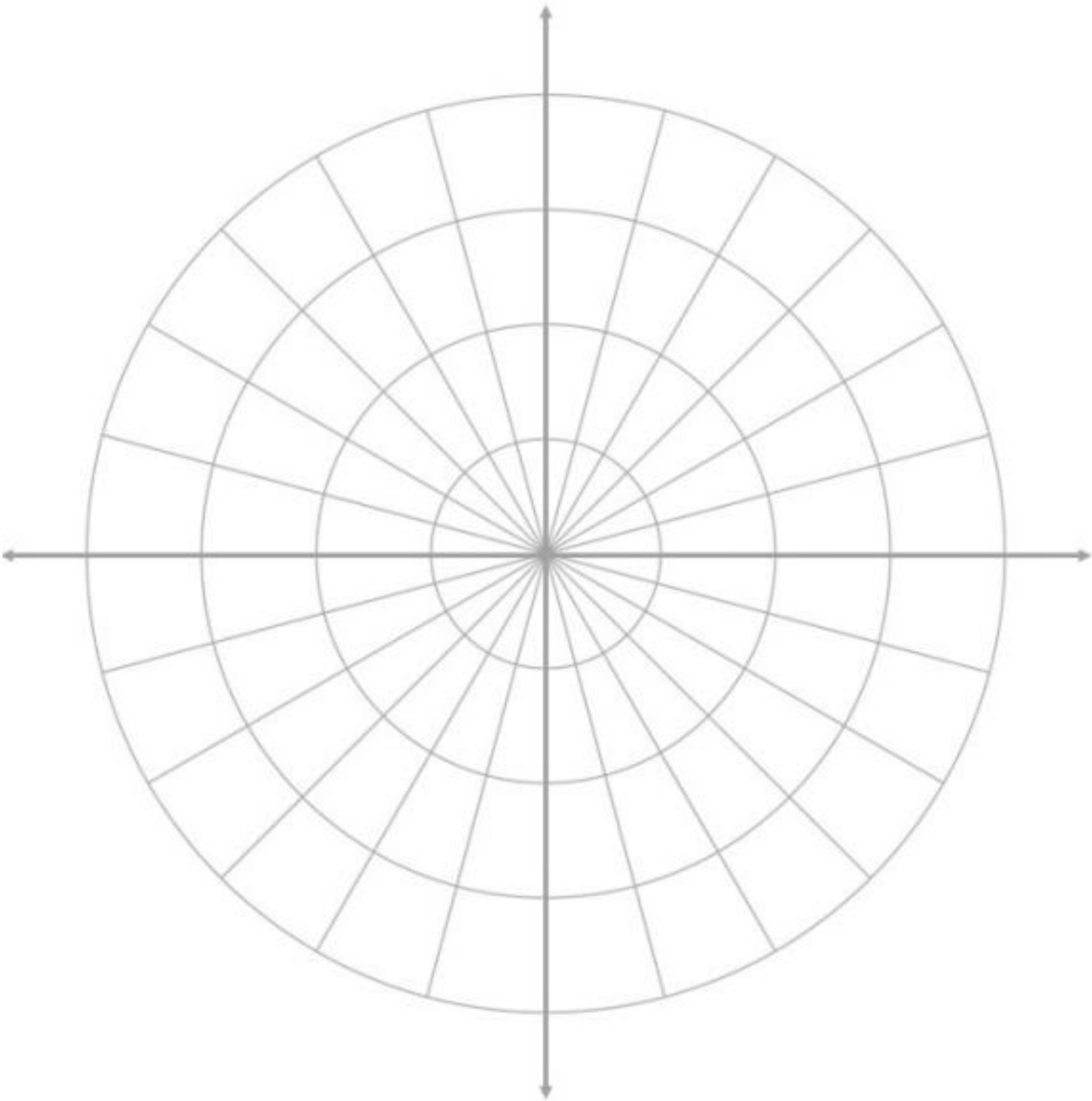


Homework

Create a table & graph the following equation:

$$r = \frac{\sin\theta\sqrt{|\cos\theta|}}{\sin\theta + 1.4} - 2\sin\theta + 2$$

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
r													



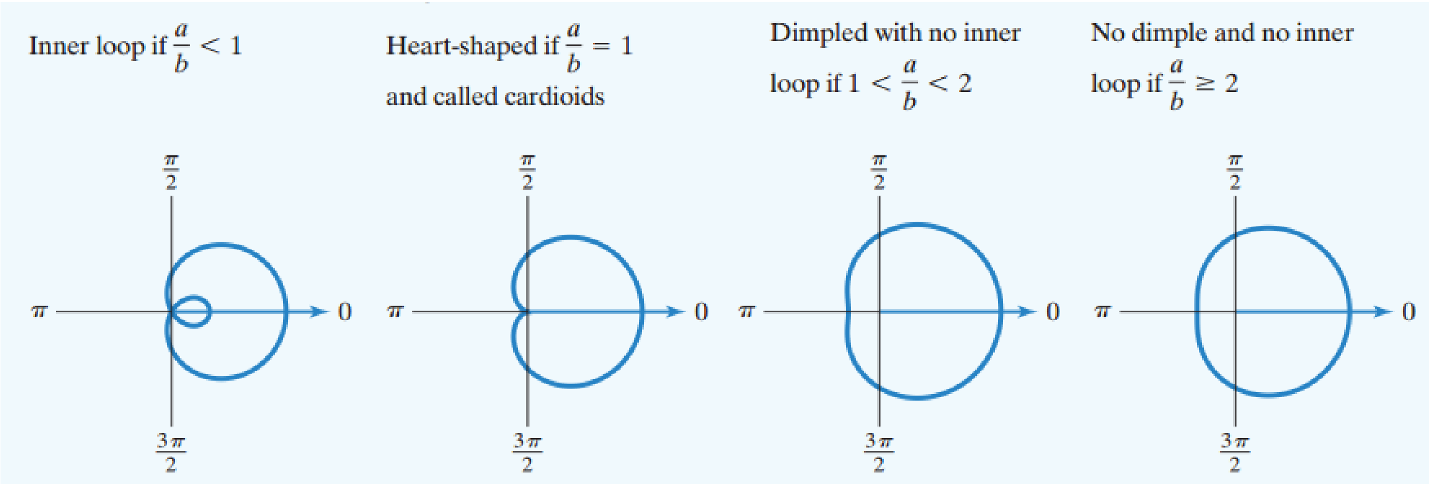
Name: ______________________________ Date: ______________ Period: ________

Chapter 6: Additional Topics in Trig
Topic 5: Graphs of Polar Equations

Typically, it is best to use all of the 'usual' angle values: all of the quadrantial and 30, 45, 60 sets.

Type 1: Limacons

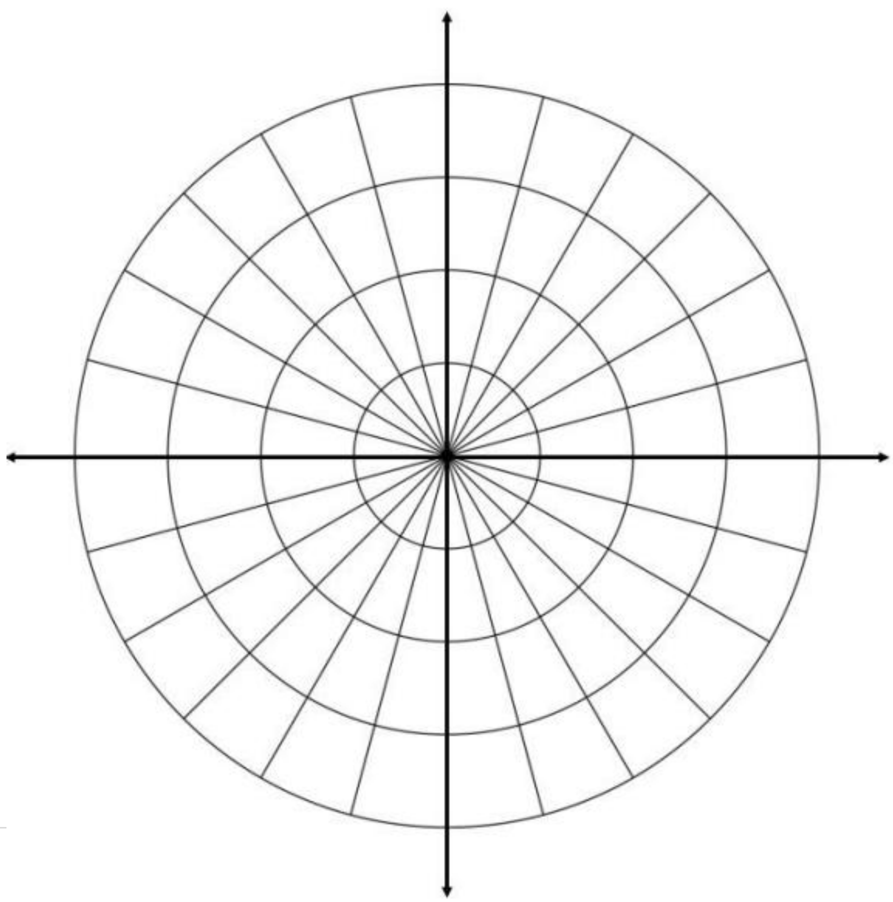
Equations of the form $r = a \pm b\sin\theta$ or $r = a \pm b\cos\theta$, where both a and b are positive, will produce these shapes. The ratio of $\frac{a}{b}$ will determine the full shape of the limacon:



θ	
0°	
15°	
30°	
45°	
60°	
75°	
90°	
105°	
120°	
135°	
150°	
165°	
180°	
195°	
210°	
225°	
240°	
255°	
270°	
285°	
300°	
315°	
330°	
345°	
360°	

Example #3: $r = 1 - \cos\theta$

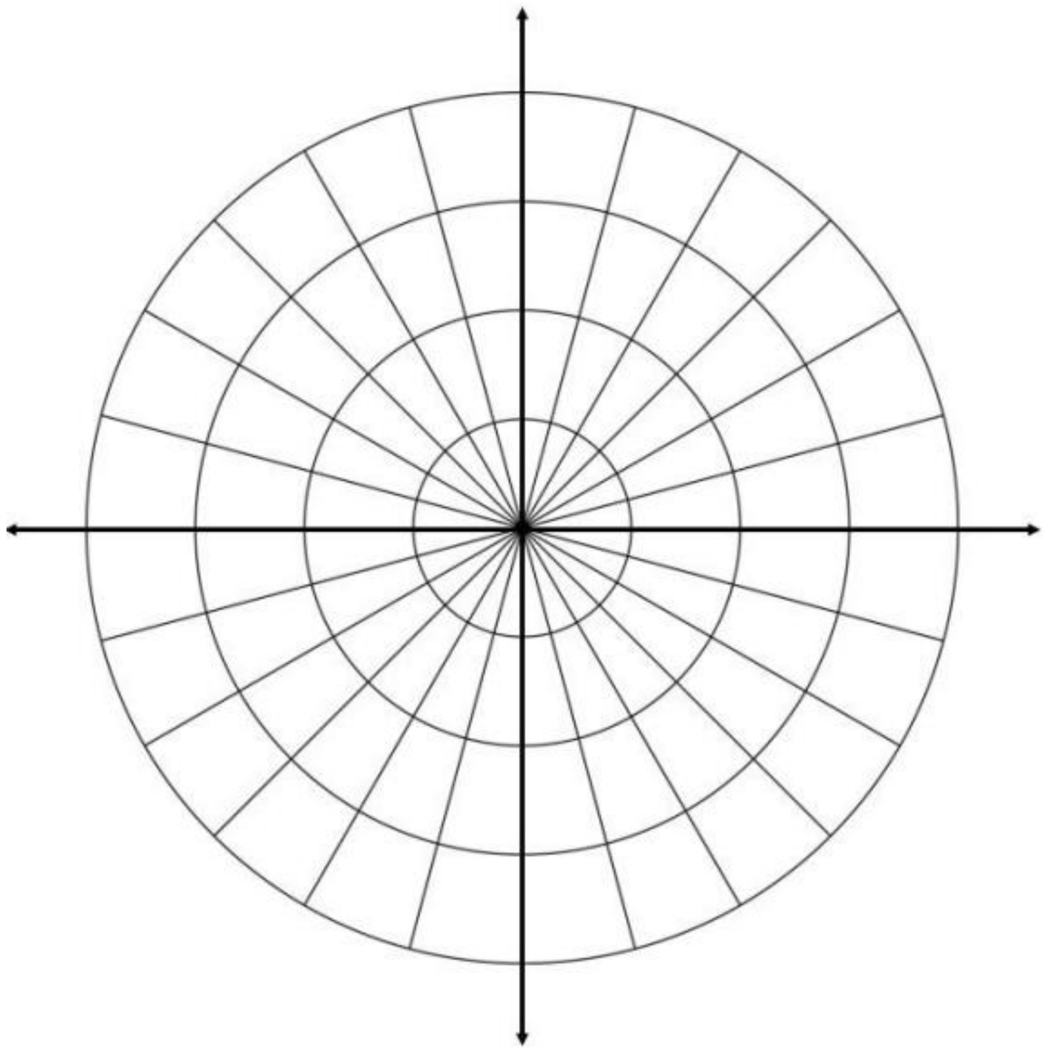
Identify the type & details from above:



Example #4: $r = 1 + 2\sin\theta$

Identify the type & details from above:

θ	
0°	
15°	
30°	
45°	
60°	
75°	
90°	
105°	
120°	
135°	
150°	
165°	
180°	
195°	
210°	
225°	
240°	
255°	
270°	
285°	
300°	
315°	
330°	
345°	
360°	



Type 2: Rose Curves

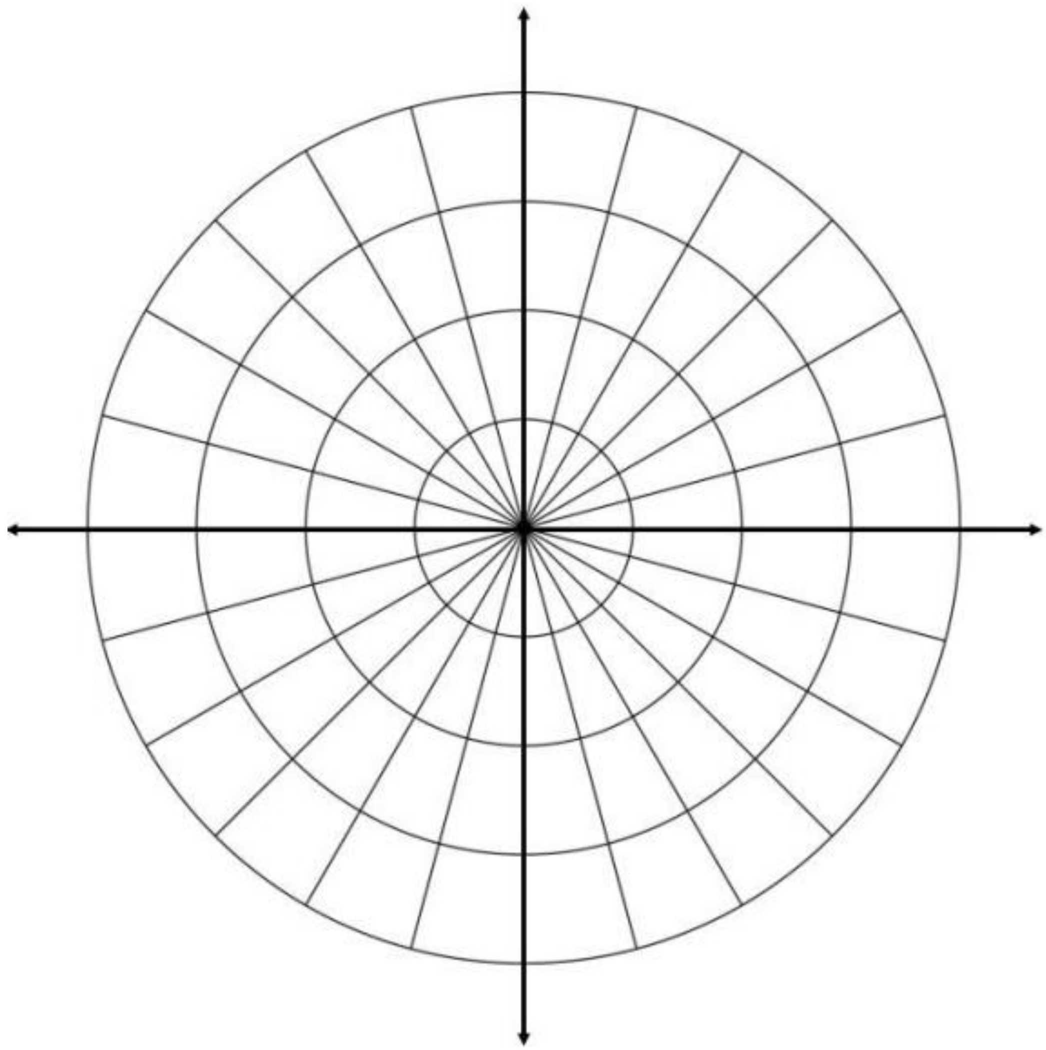
Equations of the form $\boldsymbol{r} = \boldsymbol{a}\sin\boldsymbol{n\theta}$ or $\boldsymbol{r} = \boldsymbol{a}\cos\boldsymbol{n\theta}$, where $a \neq 0$ will produce these shapes.

- If n is even, the rose will have 2n petals.
- If n is odd, the rose has n petals

Example #5: $\boldsymbol{r = 4\sin 2\theta}$

Identify the type & details from above:

θ	
0°	
15°	
30°	
45°	
60°	
75°	
90°	
105°	
120°	
135°	
150°	
165°	
180°	
195°	
210°	
225°	
240°	
255°	
270°	
285°	
300°	
315°	
330°	
345°	
360°	



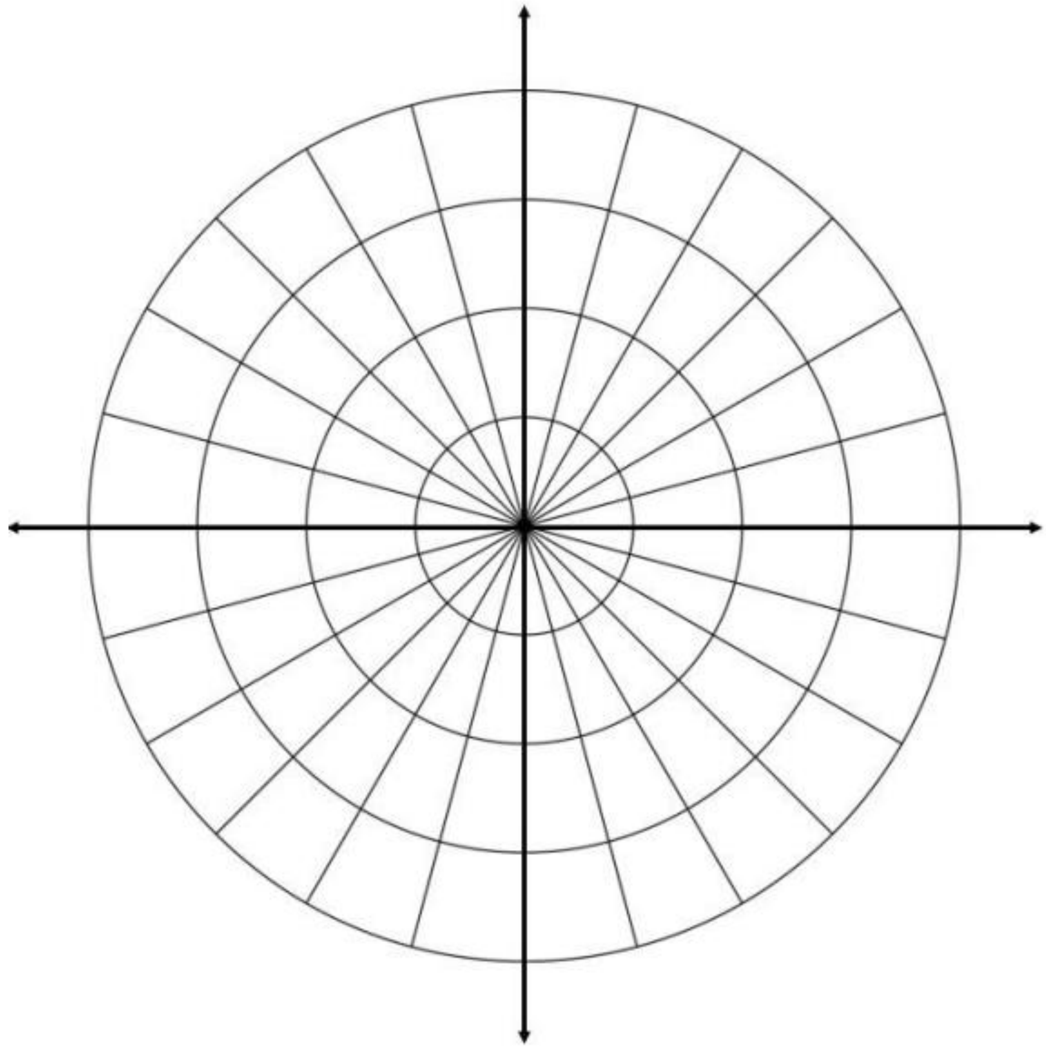
Type 3: Lemniscates

Equations of the form $r^2 = a^2 \sin 2\theta$ or $r^2 = a^2 \cos 2\theta$, where $a \neq 0$ will produce these shapes.
When creating a chart for these, remember to include (and plot) both the positive and negative values.

Example #6: $r^2 = 4 \sin 2\theta$

Identify the type & details from above:

θ	
0°	
15°	
30°	
45°	
60°	
75°	
90°	
105°	
120°	
135°	
150°	
165°	
180°	
195°	
210°	
225°	
240°	
255°	
270°	
285°	
300°	
315°	
330°	
345°	
360°	



Name: ____ Date: __________________ Period: _______

Chapter 6: Additional Topics in Trig
Topic 5: Homework

In Exercises 13–34, test for symmetry and then graph each polar equation.

13. $r = 2\cos\theta$

15. $r = 1 - \sin\theta$

17. $r = 2 + 2\cos\theta$

19. $r = 2 + \cos\theta$

21. $r = 1 + 2\cos\theta$

23. $r = 2 - 3\sin\theta$

25. $r = 2\cos 2\theta$

27. $r = 4\sin 3\theta$

29. $r^2 = 9\cos 2\theta$

31. $r = 1 - 3\sin\theta$

33. $r\cos\theta = -3$